

2

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$$

$$\lambda = 3 \quad A - 3I = \begin{bmatrix} 1-3 & 0 \\ -2 & 3-3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -2 & 0 \end{bmatrix} X = 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$-2x = 0$$

$$x = 0$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lambda = 1, \quad A - I = \begin{bmatrix} 0 & 0 \\ -2 & 2 \end{bmatrix} X = 0$$

$$-2x + 2y = 0$$

$$x = y$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$3 \begin{bmatrix} -3 & 0 & -4 \\ -6 & 2 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -3-\lambda & 0 & -4 \\ -6 & 2-\lambda & 2 \\ 0 & 0 & 6-\lambda \end{bmatrix}$$

$$-3-\lambda((2-\lambda)(6-\lambda) - (0)(2)) - 0(-6(6-\lambda) - 0) + 4(0)$$

$$-\cancel{18}(-3-\lambda)(12-8\lambda+\lambda^2) \\ = -\lambda^3 + 5\lambda^2 + 12\lambda - 36$$

$$4 \begin{bmatrix} -10 & 8 & 8 \\ -4 & 2 & 4 \\ -12 & 12 & 10 \end{bmatrix} \quad \lambda = -2, 6$$

$$\lambda = -2$$

$$\begin{bmatrix} -8 & 8 & 8 \\ -4 & 4 & 4 \\ -12 & 12 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 - x_2 - x_3 = 0$$

$$\lambda = 6$$

$$\begin{bmatrix} -16 & 8 & 8 \\ -4 & -4 & 4 \\ -12 & 12 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \begin{bmatrix} 2/3 \\ 1/3 \\ 1 \end{bmatrix}$$

$$2 \begin{bmatrix} 2/3 & 1 & 1 \\ 1/3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$5 \quad T(a_0 + a_1 t + a_2 t^2)$$

$$T(1) = T(1 + 0t + 0t^2)$$

$$= 2(1) + (4(1) - 5(0))t + (7(0) + 9(0))t^2$$

$$= 2 + 4t + 0t^2$$

$$T(t) = 2(0) + (4(0) - 5(1))t + (7(1) + 9(0))t^2$$

$$= -5t + 7t^2$$

$$T(t^2) = 2(0) + (4(0) - 5(0))t + (7(0) + 9(1))t^2$$

$$= 9t^2$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & -5 & 0 \\ 0 & 7 & 9 \end{bmatrix}$$